PARALLEL PLATE CAPACITOR

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Abstract. This document tries to give a small overview of some properties of a theoretical parallel plate capacitor. It discuss for example about the non-uniform charge repartition.

Introduction

A parallel plate (or plane) capacitor is a fundamental configuration in electrostatics, consisting of two conducting plates of area A separated by a distance d, often filled with a dielectric of permittivity ϵ. Under the application of a voltage V, the plates accumulate equal and opposite charges Q and $-Q$, creating an electric field E between them.

Charges Distribution

The idealized form of the capacitor is often assumed to have a uniform electric field given by $E = V/d$, and a capacitance C given by $C = \epsilon A/d$. However, this idealization is an oversimplification that neglects edge effects.

In practice, the mutual repulsion of like charges tends to cause a greater accumulation of charge around the edges of the plates. With more charges at the edges, the electric field between the plates will actually be more uniform than if the charges were distributed uniformly. This is because the extra charges at the edges generate additional electric field, which compensates the field that would otherwise be higher in the middle.

In a sense, the system minimizes its potential energy by adjusting the distribution of charges on the plates to create a more uniform electric field between the plates, which represents a more stable or "lower energy" configuration.

Electric Potential

The electric potential V between the plates, given by $V = -\int \vec{E} \cdot d\vec{l}$, where $d\vec{l}$ is an infinitesimal displacement along the path of integration, can no longer be simply modeled as the product of the electric field and the distance.

The non-uniform charge distribution can be modeled using advanced mathematical techniques such as solving the Laplace or Poisson equation for the electric potential.

Charging / Discharging Time

In an electrical circuit, the time constant τ for charging or discharging the capacitor, given by $\tau = RC$ where R is the resistance, will also depend on the non-uniform charge distribution. The current I, given by $I = dQ/dt$, can also be non-linear, leading to complex behaviors in transient analysis.

Test in Visualis Physics Software

In Visualis Electromagnetism it is possible to simulate/study the electric field for uniform and non-uniform charge repartition, and to observe the charge evolution.